Problem Set 3: Due Thursday Week 9

(1) From HTF: Exercise 3.2.

(2) From HTF: Exercise 3.12.

(3) From HTF: Exercise 3.29.

(4) Consider ridge regression for some fixed (x_i, y_i). Denote by $\hat{\beta}_\lambda$ the estimator resulting with λ as the penalization term. Show that $\|\hat{\beta}_\lambda\|_2$ is non-decreasing with respect to λ. Does this same property hold for the Lasso?.

(5) Let A be a positive definite $p \times p$ matrix, consider the convex function

$$f(x) = \|Ax\|_2$$

Describe the subdifferential of f at 0 in terms of A. Do the same thing for $f(x) = \|Ax\|_1$. Using this, determine the subdifferential for the weighted ℓ^1 norm arising in the grouped Lasso

$$f(x) = \lambda \sum_{k=1}^{K} \sqrt{p_k} \|x_k\|_{\ell^1}$$

where $x \in \mathbb{R}^p$ is decomposed as $x = (x_1, \ldots, x_K)$, where $x_k \in \mathbb{R}^{p_k}$ and $p = p_1 + \ldots + p_K$.

Hint: Recall how in class we discussed the subdifferential for $f_0(x) = \|x\|_1$ and $f_0(x) = \|x\|_2$ at $x = 0$. Use this information, once you have figured out how the subdifferential behave under linear change of variables.

(6) Let $f : \mathbb{R}^p \mapsto \mathbb{R}$ be a convex function and $x_0 \in \mathbb{R}^p$. If there is a non-zero slope $\beta \in \partial f(x_0)$, does it follow that x_0 cannot be a global minimum for x_0? Prove, or provide a concrete counter example.