Math 456 Spring 2018. Problem Set 3.

(1) Consider an Ehrenfest chain with N particles. Fix $i = 1, \ldots, N$. Determine $\mathrm{p}^n(i,i)$ for $n = 2, 3$. What can be said about it when n is odd?.

(2) Consider an irreducible Markov chain with N states. Explain why is it that given any two states x and y, there must be some number $k < N$ such that

$$\mathrm{p}^k(x,y) > 0.$$

Next, suppose that the chain is such that $\mathrm{p}(x,x) > 0$ for all states. Show in this case that

$$\mathrm{p}^{N-1}(x,y) > 0 \ \ \forall \text{ states } x, y.$$

(3) Write a code that takes a $N \times N$ transition probability matrix and a positive number n, and produces the n-th power of a transition probability matrix, presenting the output in visual form (i.e. writing the rows and columns of the matrix).

(a) Use this to calculate $\mathrm{p}^2, \mathrm{p}^5, \mathrm{p}^{10}, \mathrm{p}^{20}$ and p^{40} for: the Gambler's ruin (with $M = 5$) and the Ehrenfest chain (with $N = 6$).

(b) What pattern do you see as n increases for each matrix?.