

**Texas State University**  
MATH 3323: Differential Equations  
Instructor: Nestor Guillen

**Problem Set 7**

(1) Solve the IVP for each linear second order equation given below

(a)  $3\ddot{x} - \dot{x} + 2x = 0$ ,  $x(0) = 2$ ,  $\dot{x}(0) = 0$

(b)  $5\ddot{x} + 2\dot{x} + 7x = 0$ ,  $x(0) = 2$ ,  $\dot{x}(0) = 1$

(c)  $\ddot{x} + 2\dot{x} + 6x = 0$ ,  $x(0) = 2$ ,  $\dot{x}(0) = -1$

(2) Find the general solution to the following two systems

$$(a) \dot{x} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 1 & 4 \end{pmatrix} x \quad (b) \dot{x} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -4 & -1 \\ 0 & 1 & -4 \end{pmatrix} x$$

Don't forget to provide the final answer in terms of solutions that are real-valued only.

(3) For each pair of matrices  $A$  and  $\Psi(t)$  given below compute  $\dot{\Psi}(t)$  and  $A\Psi(t)$ .

(a)  $A = \begin{pmatrix} -10 & 0 \\ 0 & 5 \end{pmatrix}$ ,  $\Psi(t) = \begin{pmatrix} e^{-10t} & 0 \\ 0 & e^{5t} \end{pmatrix}$

(b)  $A = \begin{pmatrix} -10 & 0 \\ 0 & 5 \end{pmatrix}$ ,  $\Psi(t) = \begin{pmatrix} e^{-10t} & -e^{-10t} \\ 3e^{5t} & -2e^{5t} \end{pmatrix}$

(c)  $A = \begin{pmatrix} 0 & -8 & 0 \\ 8 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ ,  $\Psi(t) = \begin{pmatrix} \cos(8t) & -\sin(8t) & 0 \\ \sin(8t) & \cos(8t) & 0 \\ 0 & 0 & e^{-2t} \end{pmatrix}$

(4) (BONUS) Solve problems #3 (d),(e), and (f) from the Exam 2 Practice.

(5) (BONUS) Find the general solution to the system  $\dot{x} = Ax$ , where

$$A = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & -4 & 7 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$