Texas State University
MATH 3323: Differential Equations
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Problem Set 3

(1) Find the solution to the differential equation taking the given value for a given t

$$\text{a) } \dot{x} = \left(1 + \frac{1}{t}\right)x + t,\ x(\ln(2)) = 1,$$
$$\text{b) } \dot{x} = -\frac{2}{t}x + \frac{1}{t^2}\cos(t)\ \ x(\pi) = 0,$$
$$\text{c) } \dot{x} = \pi\cos(t)x + e^{\pi\sin(t)}\sin(t),\ x(0) = 0.$$

(2) Solve each system of equations below (i.e. if there are solutions find all of them, or else explain why there aren't any solutions)

$$\text{a) } \begin{cases} 9x_1 & -x_2 & = 3 \\ 3x_1 & +x_2 & = 1 \end{cases}$$

$$\text{b) } \begin{cases} x_1 & +x_2 & -x_3 & = 1 \\ 2x_1 & +x_2 & +x_3 & = 1 \\ x_1 & -x_2 & +2x_3 & = 1 \end{cases}$$

$$\text{c) } \begin{cases} x_1 & & -x_3 & = 0 \\ 3x_1 & +x_2 & +x_3 & = 1 \\ -x_1 & +x_2 & +2x_3 & = 2 \end{cases}$$

(3) For each family of vectors, determine wether the vectors are linearly independent or not, and in case they are linearly dependent, find a linear relation between them.

a) $\mathbf{x}_1 = (2, 2, 0),\ \ \mathbf{x}_2 = (0, -2, 2),\ \ \mathbf{x}_3 = (1, 0, 1)$
b) $\mathbf{x}_1 = (2, 1, 0),\ \ \mathbf{x}_2 = (0, 1, 0),\ \ \mathbf{x}_3 = (-1, 2, 0)$
c) $\mathbf{x}_1 = (1, 1, 0, 0),\ \ \mathbf{x}_2 = (0, 1, 1, 0),\ \ \mathbf{x}_3 = (0, 0, 1, 1),\ \ \mathbf{x}_4 = (0, 0, 0, 1)$

(4) Consider the two vector valued functions

$$\mathbf{x}_1(t) = (2e^t, 3) \text{ and } \mathbf{x}_1(t) = (4, 6e^{-t}).$$

For any given fixed value t_0, show that the two dimensional vectors $\mathbf{x}_1(t_0)$ and $\mathbf{x}_2(t_0)$ are linearly dependent. At the same time, show that $\mathbf{x}_1$ and $\mathbf{x}_2$ as functions of t are linearly independent.

(5) (BONUS) Determine a function $c(t)$ so that if we define the function

$$x(t) = e^{5t}c(t)$$

Then x solves

$$\dot{x} = 5x + \sin(t).$$

Discuss any similarities or relationship to one of the methods to solve differential equations discussed in class.

(6) (BONUS) Let f be a real function of a single real variable with the property that for some constant $L > 0$ we have that

$$|f(x) - f(y)| \leq L|x - y| \text{ for all numbers } x, y.$$

Show that given an initial value α, the problem

$$\dot{x} = f(x), \quad x(0) = \alpha$$

cannot have two different solutions.

Hint: Assume $x_1(t)$ and $x_2(t)$ are two solutions, and prove that $(x_1(t) - x_2(t))^2$ satisfies a differential inequality like the one in problem 5 from Problem Set 2.