Texas State University
MATH 3323: Differential Equations
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Problem Set 1

(1) Compute each indefinite integral

$$\int \frac{1}{1-x^2}\,dx$$
$$\int \frac{x}{1+x^2}\,dx$$
$$\int \frac{\sin(x)}{5-2\cos(x)}\,dx$$

(2) For each case, find the solution $x(t)$ for the differential equation which has the given value

$$\dot{x}(t) = 2x(t) + e^{2t}\cos(t), \;\; x(2) = 1$$
$$\dot{x}(t) = -10x(t) - e^{-10t}t^2, \;\; x(0) = 0$$
$$\dot{x}(t) = x(t) + 1, \;\; x(10) = -1$$

(3) Compute $\frac{df}{dx}$ for each given f

$$f(x) = x^4 + x^3 + x^2 + x + 1$$
$$f(x) = \cos(3x) + \cos(2x) + \cos(x) + 1$$
$$f(x) = \frac{\sin(x^2)}{2+\cos(x)} - \frac{(\sin(x))^2}{2+\cos(x+2)}$$

(4) Find the value α such that if $x(t)$ solves the initial value problem

$$\dot{x} = -\tfrac{2}{3}x + 1 - \tfrac{1}{2}t, \; x(0) = \alpha$$

then $x(t)$ does not change sign but takes the value $x(t) = 0$ for at least some t.

(5) (BONUS) Let $x(t)$ be a positive function satisfying the **inequality**

$$\dot{x}(t) \leq -\lambda x(t) \;\text{ for all } t$$

for some number $\lambda > 0$. Show that

$$x(t) \leq x(0)e^{-\lambda t} \text{ for } t > 0.$$

Discuss: what is the relationship between the value of λ and the behavior of $x(t)$ as t goes to infinity? (for example, take $x_1(t)$ and $x_2(t)$ functions as above with two different constants λ_1 and λ_2, what can you say about $x_1(t)/x_2(t)$ as $t \to \infty$ if $\lambda_1 > \lambda_2$?).