

### Math 3323 Exam 3 Practice

(1) Find a fundamental matrix for  $\dot{x} = Ax$  for each  $A$  given below

$$\begin{array}{ll} \text{(a)} \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} & \text{(b)} \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} \\ \text{(c)} \begin{pmatrix} 3 & 0 \\ 1 & 7 \end{pmatrix} & \text{(d)} \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}. \end{array}$$

(2) Find at least one solution to each of the following inhomogeneous problems

$$\begin{array}{ll} \text{(a)} \ddot{x} - 8x = t^2 - 1 & \text{(b)} \ddot{x} - 8x = 3t \\ \text{(c)} \ddot{x} + 9x = \cos(3t) & \text{(d)} \ddot{x} + 9x = \cos(3t)^2 \\ \text{(e)} \ddot{x} - 2\dot{x} - x = \cos(t) & \text{(f)} \ddot{x} - 2\dot{x} - x = \sin(t). \end{array}$$

(3) Solve each IVP below

$$\begin{array}{l} \text{(a)} \dot{x} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} te^t \\ 7 \end{pmatrix}, x(0) = \begin{pmatrix} -1 \\ -2 \end{pmatrix} \\ \text{(b)} \dot{x} = \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix} x + \begin{pmatrix} 0 \\ -\sin(t) \end{pmatrix}, x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \text{(c)} \dot{x} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} e^t \\ 0 \end{pmatrix}, x(0) = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \\ \text{(d)} \dot{x} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} x + \begin{pmatrix} e^t \\ 0 \end{pmatrix}, x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \end{array}$$

(4) For each of the inhomogeneous equations in Problem #2, find the solution with initial conditions  $x(0) = 0$  and  $\dot{x}(0) = 1$ .