

### Math 3323 Exam 2 Practice

- (1) For each matrix below, write the characteristic polynomial, find all its eigenvalues, and compute one eigenvector for each eigenvalue.

$$(a) \begin{pmatrix} 9 & 3 \\ 3 & 9 \end{pmatrix} \quad (b) \begin{pmatrix} 10 & 4 \\ 0 & 5 \end{pmatrix} \quad (c) \begin{pmatrix} 9 & -6 \\ 12 & -3 \end{pmatrix}$$

$$(d) \begin{pmatrix} 1 & 10 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 8 \end{pmatrix} \quad (e) \begin{pmatrix} -1 & 0 & 0 \\ 0 & 4 & 20 \\ 1 & 16 & 8 \end{pmatrix} \quad (f) \begin{pmatrix} 7 & 4 & 5 \\ 0 & 3 & 11 \\ 0 & 0 & 2 \end{pmatrix}$$

- (2) For each matrix in problem #1, write down the solutions corresponding to the eigenvectors you computed, and use this to write a general solution to the corresponding system  $\dot{x} = Ax$ .
- (3) For each matrix in problem #1 solve the corresponding system  $\dot{x} = Ax$  with the initial conditions given below:

(For (a), (b), (c) in Problem #1) :

$$(a) \begin{pmatrix} 6 \\ 9 \end{pmatrix} \quad (b) \begin{pmatrix} 7 \\ 31 \end{pmatrix} \quad (c) \begin{pmatrix} -4 \\ 13 \end{pmatrix}$$

(For (d), (e), (f) in Problem #1) :

$$(d) \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \quad (e) \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix} \quad (f) \begin{pmatrix} -4 \\ 15 \\ 0 \end{pmatrix}$$

- (4) Write each of the following second order linear equations as a first order system and write down the corresponding characteristic polynomial and its eigenvalues.

$$(a) \ddot{x} - 2\dot{x} - 24x = 0 \quad (b) \ddot{x} + 11\dot{x} + 30x = 0 \quad (c) \ddot{x} - 16\dot{x} + 39x = 0 \\ (d) \ddot{x} + 6\dot{x} + 8x = 0 \quad (e) \ddot{x} - 100x = 0 \quad (f) \ddot{x} - 42\dot{x} + 425x = 0$$

- (5) For each second order equation in Problem #4 find the solution with initial condition

$$x(0) = 1, \dot{x}(0) = 0,$$

and the solution with initial condition

$$x(0) = 0, \dot{x}(0) = 1.$$