Math 3323 Exam 1 Practice
(Updated 09/28/2019)

(1) For each equation below, say which method is well suited to find its solutions (there could be more than one!), and then use it to find a formula for the general solution to the equation

(a) $y' = (2 + \cos(x))y$	(e) $y' = (1 + x)y(100 - y)$
(b) $y' = \cos(x)y$	(f) $y' = 3xy + x$
(c) $y' = \frac{\sin(t)}{2+\cos(t)}y$	(g) $y' = 2y^2$
(d) $y' = -5y^2$	(h) $y' = 9y + 4x$

(2) Consider the differential equation

$$\frac{dy}{dx} = (y - 1)(y - 2)$$

Then,

(a) Use separation of variables to find a general formula for a solution $y(x)$ that is written in terms of the initial value $y(0)$.

(b) Provide two examples of solutions to differential equation which are constant.

(c) Determine what happens with $y(x)$ as $x \to \infty$ in these two cases 1) when $1 < y(0) < 2$ and when $y(0) > 2$.

(3) Consider the differential equation

$$\dot{x} = \frac{x(x - 2)}{(x + 2)t^2}$$

and find a solution such that $x(0) = 0$ and find a solution such that $x(0) = 2$.

(4) Consider the first order linear differential equation

$$\dot{x} - \tfrac{2}{1+t}x = 0.$$

(a) Find all solutions to this equation, writing the undetermined parameter C in terms of the initial value, $x(0)$.

(b) Find the solution to the equation

$$\dot{x} - \tfrac{2}{1+t}x = (1 + t)^2 \sin(t),$$

with initial value $x(0) = 0$.

(c) By adding up the solutions obtained in (a) and (b), find a solution to the second differential equation taking the value 3 at $t = 0$.

(5) Pair each function on the left column with the linear system it solves on the right column

i) $\mathbf{x}(t) = \begin{pmatrix} e^{4t} \\ e^{4t} \end{pmatrix}$ a) $\dot{x} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} x$

ii) $\mathbf{x}(t) = \begin{pmatrix} 4e^{-t} \\ 3 \end{pmatrix}$ b) $\dot{x} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} x$

iii) $\mathbf{x}(t) = \begin{pmatrix} t + \frac{1}{2}(e^{2t} - 1) \\ \frac{1}{3}(e^{3t} - 1) \end{pmatrix}$ d) $\dot{x} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} x$

iv) $\mathbf{x}(t) = \begin{pmatrix} \cos(t+1) \\ \sin(t+1) \end{pmatrix}$ c) $\dot{x} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} x + \begin{pmatrix} 2 - 2t \\ 1 \end{pmatrix}$

(6) The functions

$$x_1(t) = e^{9t} \text{ and } x_2(t) = e^{-3t}$$

both solve the second order linear differential equation

$$\ddot{x} - 6\dot{x} - 27x = 0.$$

Using this information, find a solution x to the above equation for each of the following initial conditions

(a) $x(0) = 0,\ \dot{x}(0) = 1$ (d) $x(0) = -11,\ \dot{x}(0) = 8$
(b) $x(0) = 3,\ \dot{x}(0) = 1$ (e) $x(0) = 10,\ \dot{x}(0) = 23$
(c) $x(0) = -13,\ \dot{x}(0) = 59$ (f) $x(0) = 8,\ \dot{x}(0) = 4$