

Texas State University
MATH 3323: Differential Equations
Instructor: Nestor Guillen

Problem Set 6

This problem set is due Thursday, March 12th. It covers the material of this past week and Section 7.2 of the book.

- (1) For each matrix bellow, find all vectors such that $Ax = 0$.

a) $A = \begin{pmatrix} 2 & 0 \\ 3 & 0 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ c) $A = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$

- (2) Compute the determinant of each of the matrices in the previous problem. Observe which matrices have $\det(A) = 0$ and which have $\det(A) \neq 0$ and compare it with the size of the set of solutions to $Ax = 0$.

- (3) Find the eigenvalues of the following matrices

a) $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ b) $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$

- (4) Consider the matrix-valued functions of t

$$\mathbf{A}(t) = \begin{pmatrix} e^t & 2e^{-t} \\ 2e^{-t} & 1 \end{pmatrix} \quad \mathbf{B}(t) = \begin{pmatrix} 1 & 0 \\ 0 & e^{-t} \end{pmatrix}$$

Then, compute the following expressions

a) \mathbf{AB} c) $\frac{d}{dt}(\mathbf{AB})$
b) $\frac{d}{dt}\mathbf{A}$ d) $(\frac{d}{dt}\mathbf{A})\mathbf{B} + \mathbf{A}(\frac{d}{dt}\mathbf{B})$

- (5) (BONUS) Given differentiable functions $a_{11}(t), a_{12}(t), a_{21}(t)$, and $a_{22}(t)$, compute the derivative of

$$\det \begin{pmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{pmatrix}$$

and show the result coincides with the sum

$$\det \begin{pmatrix} \dot{a}_{11}(t) & a_{12}(t) \\ \dot{a}_{21}(t) & a_{22}(t) \end{pmatrix} + \det \begin{pmatrix} a_{11}(t) & \dot{a}_{12}(t) \\ a_{21}(t) & \dot{a}_{22}(t) \end{pmatrix}$$

- (6) (BONUS) Let x be a real number, find the limit of the sequence

$$\lim_{n \rightarrow \infty} \frac{x^n}{n!}$$

(remember that $n!$ denotes the product $n(n-1)(n-2)\dots 1$). Based on your answer, determine the set of values of x for which the following series converges and is finite

$$\sum_{n=0}^{\infty} \frac{1}{n!} x^n$$