Texas State University
MATH 3323: Differential Equations
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Problem Set 4 (updated)

This problem set covers a bit more of integrating factor and then shifts to systems, the relevant sections are 2.2, 7.1, and 7.2.

(1) Consider the differential equation

$$\frac{dy}{dx} = 4y + 3e^{8x}(x+1)$$

Find a general formula for solutions of the equation. Then, find a solution with initial value $y(0) = 0$.

(2) Find an explicit solution to each of the following equations taking the prescribed value

(a) $\dot{x} = \sin(t)x + e^{-\cos(t)}\sin(t), \quad x(0) = -1$

(b) $\dot{x} = 2tx, \; x(0) = 1$

(c) $\dot{x} = \frac{2}{t}x, \; x(1) = 1$

(d) $\dot{x} = \frac{3}{t}x + 5t, \; x(1) = 1.$

(3) Consider the function given by the integral formula

$$x(t) = e^t \int_0^t e^{-s}\sin(s)\, ds$$

Find a differential equation of which $x(t)$ is a solution.

Hint: You may compute the definite integral explicitly to answer this quesiton, but this is not necessarily the only way to find the answer.

(4) Consider the following list of functions

(a) $x(t) = 2\cos(5t) - 3\sin(5t)$

(b) $x(t) = e^{2t}\cos(t)$

(c) $x(t) = e^{-t} + e^t + t$

and match each function to the differential equation it solves in the following list

(i) $\ddot{x} = x - t$

(ii) $\ddot{x} - 4\dot{x} + 5x = 0$

(iii) $\ddot{x} = -25x$

(5) (BONUS) Find the value α such that if $x(t)$ solves the initial value problem

$$\dot{x} = -\tfrac{2}{3}x + 1 - \tfrac{1}{2}t, \; x(0) = \alpha$$

then $x(t)$ does not change sign but takes the value $x(t) = 0$ for at least some t.

(6) (BONUS) Consider the nonlinear differential equation

$$\dot{x} = \frac{1}{3}x\left(\frac{7}{23} - x\right)$$

Then

(a) Find the general formula for the solution (in terms of the initial value $x(0)$).
(b) When $x(0) = 1$, what happens with $x(t)$ as $t \to \infty$?
(c) Find a solution $x(t)$ of the equation which is constant in time.
(d) Give an example of initial value $x(0)$ so that $\lim_{t\to\infty} x(t) = 0$.