Texas State University
MATH 3323: Differential Equations, Spring 2020
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Problem Set 2

This problem set is concerned mostly with more calculus review and first order linear equations. The new relevant method is integrating factor (Section 2.1).

(1) Match each function on the left column with the differential equation it solves on the right column.

a) $x(t) = 4e^{-2t}$	A) $\dot{x} = \cos(t)x$
b) $x(t) = e^{\sin(t)+2}$	B) $\dot{x} = 2x + e^{2t}$
c) $x(t) = e^{2t}t$	C) $\dot{x} = \frac{x}{t+1}$
d) $x(t) = t + 1$	D) $\dot{x} = -2x$

(2) Use the product rule and chain rules to compute the derivative $\dot{x}$ of each of the following functions

(a) $x(t) = e^{4t}(t + t^2)$
(b) $x(t) = e^{-\ln(t+1)}(\sin(2t) + t)$
(c) $x(t) = 2e^{3t} + e^{3t}\int_0^t e^{-3s}s\,ds$
(d) $x(t) = e^{-t^2/2}\sin(t)$

(3) In each case find the function $y(x)$ or $x(t)$ having the given derivative and taking the indicated value at 0

(a) $y'(x) = \cos(\pi x + \pi)$ and $y(0) = 0$
(b) $\dot{x}(t) = 1$ and $x(0) = -1$
(c) $\dot{x}(t) = -2t$ and $x(0) = 10$
(d) $y'(x) = 1/(1 + x^2)$ and $y(0) = \pi/2$

(4) Find the solution to the differential equation taking the given value for a given t

$$\text{a) } \dot{x} = \left(1 + \frac{1}{t}\right)x + t,\ x(\ln(2)) = 1,$$

$$\text{b) } \dot{x} = -\frac{2}{t}x + \frac{1}{t^2}\cos(t)\ \ x(\pi) = 0,$$

$$\text{c) } \dot{x} = \pi\cos(t)x + e^{\pi\sin(t)}\sin(t),\ x(0) = 0.$$

(5) (BONUS) You are given a function $y(x)$ and all you know about it is that

$$e^{-x^2}y'(x) - 2xe^{-x^2}y(x) = 0.$$

Show that $y(x)/e^{x^2}$ is independent of x, and conclude from here that

$$y(x) = e^{x^2}y(0).$$

(6) (BONUS) Determine a function $c(t)$ so that if we define the function

$$x(t) = e^{5t}c(t)$$

Then x solves

$$\dot{x} = 5x + \sin(t).$$

Discuss any similarities or relationship to one of the methods to solve differential equations discussed in class.