Texas State University
MATH 3323: Differential Equations
Instructor: Nestor Guillen

Problem Set 11 (Bonus Problem Set)

(1) Find the Laplace transform of each given function (do not forget to show your work, most of these calculations follow by a straightforward calculation, but you may use any of the identities on table 6.2.1 of the textbook or any of the identities given in class)

(a) $f(t) = te^{2t} - t^2e^{-4t}$

(b) $f(t) = 2t^2 + 2t + 4$

(c) $f(t) = e^{2t}\sin(4t) + \cos(t)$

(d) $f(t) = h_1(t) - h_2(t)$

Note: The function $h_\alpha(t)$ is the function which is equal to 0 as long as $t < \alpha$ and equal to 1 as long as $t \geq 1$, it is denoted by u_c in the book (c takes the place of α).

(2) Find the inverse Laplace transform of the following functions

(a) $F(s) = \dfrac{5!}{(s-4)^5}$

(b) $F(s) = \dfrac{1-2s}{s^2+4s+5}$

(c) $F(s) = \dfrac{e^{-s}+e^{-4s}}{s}$

(d) $F(s) = \dfrac{e^{-2s}}{s^2+s-2}$

(3) Solve the following initial value problems

(a) $y'' - 4y = e^t + e^{-t}, \;\; y(0) = 1, \; y'(0) = 0$

(b) $y'' + 100y = \sin(4t) + \cos(4t), \; y(0) = 0, \; y'(0) = 1$

(4) A function $f : \mathbb{R} \to \mathbb{R}$ is said to be periodic with period T if T is a number such that $f(t+T) = f(t)$ for all $t \in \mathbb{R}$. Show that if f is periodic with period T then

$$\mathcal{L}(f(t))(s) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$$

Use this to compute the Laplace transform $\mathcal{L}(f)$ of the function f given by

$$f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & 1 \leq t < 2, \end{cases} \quad \text{with } f(t+2) = f(t) \text{ for all } t.$$