Texas State University
MATH 3323: Differential Equations
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Problem Set 10

This problem set deals with second order equations, concretely the definition of the Wronkskian (Section 3.2), the method of undetermined coefficients (Section 3.5), and variation of parameters (3.6). It is due Tuesday April 28th.

(1) For the differential equation and pairs of numbers λ_1 and λ_2 given in each case bellow, find numbers A and B such that the function

$$Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

is a solution to the inhomogeneous equation.

(a) $\ddot{x} + \dot{x} = e^{2t} + e^{-2t},\ \lambda_1 = 2,\ \lambda_2 = -2$
(b) $\ddot{x} + x = 3e^{2t} - e^{3t},\ \lambda_1 = 2,\ \lambda_2 = 3$
(c) $\ddot{x} + 2\dot{x} + 6x = \sqrt{5}e^{-t} + 1,\ \lambda_1 = -1,\ \lambda_2 = 0.$

(2) For the differential equation and frequency ω given in each case bellow, find numbers A and B such that the function

$$A\cos(\omega t) + B\sin(\omega t)$$

is a solution to the inhomogeneous equation.

(a) $3\ddot{x} - \dot{x} + 2x = 3\cos(2t),\quad \omega = 2$
(b) $5\ddot{x} + 2\dot{x} + 7x = \sin(3t) + \cos(3t),\quad \omega = 3$
(c) $\ddot{x} + 2\dot{x} + 6x = -11\cos(4t) + 3\sin(4t),\quad \omega = 4.$

(3) Use variation of parameters to find the solution to each IVP

(a) $\ddot{x} - 2x = 3t,\ x(0) = 0, \dot{x}(0) = 1$
(b) $\ddot{x} - 2\dot{x} - x = 2e^{-5t} - e^{-7t},\ x(0) = 3, \dot{x}(0) = -1.$
(c) $\ddot{x} - 2\dot{x} - x = 2t - 1,\ x(0) = -3, \dot{x}(0) = 1$

(4) (BONUS) Let u_1 and u_2 be a fundamental family of solutions to the equation

$$a\ddot{u} + b\dot{u} + cu = 0$$

Let $W(t)$ denote the Wronskian of u_1 and u_2. Show that W solves the differential equation

$$\dot{W} = -\frac{b}{a}W$$

Interpret the sign of b/a (and thus whether $W(t)$ grows or decays exponentially) in the physical models where -$b\dot{u}$ represents a damping force.

(5) (BONUS) For each equation bellow, find the solution to the problem with initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$,

(a) $\ddot{x} + \dot{x} + x = e^t$

(b) $\ddot{x} + \dot{x} + x = e^{2t}$

(c) $\ddot{x} + \dot{x} + x = e^{3t}$

(d) $\ddot{x} + \dot{x} = e^{3t} + e^{2t} + e^t$

(6) (BONUS) For each equation bellow, find the solution to the problem with initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$,

(a) $\ddot{x} + \dot{x} + x = \cos(t)$

(b) $\ddot{x} + \dot{x} + x = \sin(t)$

(c) $\ddot{x} + \dot{x} + x = \cos(2t)$

(d) $\ddot{x} + \dot{x} = 2\cos(t)^2 + 4\sin(t) - 1$